MATHEMATICAL COMPETITIVE EXAMINATION - II

Unit - I :-

Chain Rule, Time and Work.

Unit - II :-

Time and Distance

Unit - III :-

Problems On Train

Unit - IV :-

Boats and Stream.

Unit - V :-

Allegation or Mixture.

Unit - I.

Chain Rule :-

Direct Proportion :-

Two quantities are said to be directly proportional, if on the increase (or decrease) of the one the other increases (or decreases) to the same extent.

Example :- i). cost is directly proportional to the number of articles.

ii) work done is directly proportional to the number of man working on it.

Indirect Proportion :-

Two quantities are said to be indirectly proportional, if on the increase of the one the other decreases to the same extent and vice versa.

Example : i) A time taken by a car is covering a certain distance is inversely proportional to the speed of the car.

ii) Time taken to finish a work is inversely proportional to the number of persons working at it.

Chain Rule Formula :-

Men = M ; Days = D ; Time / hours = T & work

I

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$$

Note that : W_2 is left side and W_1 is on right

Problem :

1) A canteen requires 105 kgs of wheat for a week. How many kgs of wheat will it require for 58 days?

Soln :

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_2.$$

$D_1 = 7$ days, $W_1 = 105$ kgs

$D_2 = 58$ days ; $W_2 = ?$

Let the required quantity x kgs.

$$D_1 W_2 = D_2 W_1.$$

$$7 \times x = 105 \times 58.$$

$$x = 15 \times 58.$$

$$\begin{array}{r} 4 \\ 58 \times 15 \\ \hline 290 \\ 58 \\ \hline 870 \end{array}$$

$$x = 870 \text{ kgs}.$$

Hence the canteen will require 870 kgs of wheat for 58 days.

2). If 36 men can do a piece of work in 25 hours. in how many hours will 15 men do it ?.

Soln:

$M_1 = 36$ men. , $T_1 = 25$ hours.

$M_2 = 15$ men. , $T_2 = x$.

$M_1 T_1 W_2 = M_2 T_2 W_1$.

Let the required number of hours be x

$36^{12} \times 25 \times 1 = 15^{3} \times x \times 1$

$x = 60$ hrs.

∴ Hence 15 men can do it in 6 hrs.

3). 35 women can do a piece of work in 15 days. & How many women could be required to do the same work in 25 days

Soln:

$M_1 = 35$, $D_1 = 15$.

$M_2 = x$, $D_2 = 25$.

$M_1 D_1 W_2 = M_2 D_2 W_1$

$35^{7} \times 15^{3} \times 1 = x \times 25^{5} \times 1$

$x = 21$

∴ Hence 21 women can do it in 25 days

4) A certain number of people supposed to complete the work in 24 days. The work however took 32 days. Since 9 people were absent throughout. How many people were supposed to be working originally?

Soln:

$D_1 = 24$ days, $W_1 = 1$

$D_2 = 32$ days, $W_2 = 1$.

$M_1 = x$. $M_2 = x - 9$.

$M_1 D_1 W_2 = M_2 D_2 W_1$

$x \cdot 24 \cdot 1 = (x-9) 32 \cdot 1$

$24x = 32x - 288$

$24x - 32x = -288$

$-8x = -288$

$x = 36$.

5) If 5 students utilize 18 pencils in 9 days, how long at the same rate will 66 pencils last for 15 students? (11)

Soln:

$M_1 = 5$, $M_2 = 15$.

$W_1 = 18$, $W_2 = 66$.

$D_1 = 9$, $D_2 = x$

Let the required number of days be x

$M_1 D_1 W_2 = M_2 D_2 W_1$

$5 \times 9 \times 66 = 15 \times x \times 18$.

$33 = 3x$

$x = 33/3$

$x = 11$.

∴ 11 days for required for 15 student to utilize 66 pencils

6) If 20 men can build a wall 56 meters long. in 6 days, what length of a similar wall can be built by 35 men in 3 days ? (149)

Soln:-

$M_1 = 20$ $M_2 = 35$

$W_2 = ?$ $W_1 = 56$.

$D_1 = 6$. $D_2 = 3$.

$M_1 D_1 W_2 = M_2 D_2 W_1$.

Let the required length be x.

$20 \times x \times 6 = 35 \times 56 \times 3$

$x = 7 \times 7$

$x = 49$.

∴ 49 meter wall can be built by 35 men in 3 days.

7) 8 man working for 9 hrs a day complete a piece of work in 20 days. In how many days can 7 man working for 10 hrs a day complete the same piece of work ?

Soln:

$M_1 = 8$ $\quad M_2 = 7$.

$T_1 = 9$ hrs $\quad T_2 = 10$.

$D_1 = 20$ day $\quad D_2 = ?$

$W_1 = 1$, $W_2 = 1$.

Let the required number of days.

$W_2 M_1 T_1 D_1 = M_2 D_2 T_2 W_1$.

$\therefore 1 \times 8 \times 9 \times 20 = 7 \times 10 \times x$

$7x = 144$.

$x = \frac{144}{7}$

$x = 20\frac{4}{7}$.

The required no. of days is $20\frac{4}{7}$

8 If 12 men or 18 women can do a work in 14 days. then in how many days will 8 men and 16 women do the same work?

Soln:

12 men = 18 women.

8 Men $= \left(\frac{18}{12} \times 8\right)$

$= 12$ women.

8 men and 16 women $= 12 + 16 = 28$ women.

$M_1 = 18$ women $\quad M_2 = 28$ women.

$D_1 = 14$ days $\quad D_2 = x$.

$W_1 = 1$ $\quad W_2 = 1$.

Let the required no. of days

$M_1 D_1 W_2 = M_2 D_2 W_1$.

$18 \times 14 = 28 \times x$.

$x = 9$

∴ The required number of days is 9.

a) Five press compositors can set 625 pages of a manuscript in 16 days of $10\frac{1}{2}$ hours each: each page has 60 lines and each line has 40 letters. In how many days of 8 hours each, will 10 compositors set 1000 pages of a manuscript, each page having 45 lines with 50 letters in each line?

Soln:

$M_1 = 5$. $M_2 = 10$.

$D_1 = 16$ days, $T_1 = \frac{21}{2}$. $T_2 = 8$.

$W_1 = 625 \times 60 \times 40$. $W_2 = 1000 \times 45 \times 50$

$D_2 = ?$

Let the required number of days be x. Then

$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$.

$5 \times 16 \times \frac{21}{2} \times 1000 \times 45 \times 50 = 10 \times 8 \times x \times 625 \times 60 \times 40$

$x = \frac{5 \times 16 \times 21 \times 1000 \times 45 \times 50}{10 \times 8 \times 625 \times 60 \times 40}$

$x = 6\frac{3}{4} \Rightarrow x = 15\frac{3}{21}$.

Hence required number of days $= 15\frac{3}{4}$.

10) Rocky can walk a certain distance in 40 days. When he rests 9 hours a day. How long will he takes to walk twice the distance, twice as fast and rest twice as long each day?

Soln:

Let the distance in the two cases be y and $2y$ respectively.

Let the speed in the two cases be $2z$ and z respectively.

Let the required number of days be x.

$M_1 = 1$ $\quad M_2 = 1$

$D_1 = 40$ days $\quad D_2 = ?$

$W_1 = y \times 2z \times 9$. $\quad W_2 = 2y \times z \times 18$

Then.

$$M_1 D_1 W_2 = M_2 D_2 W_1.$$

$$1 \times 40 \times 2y \times z \times 18 = 1 \times x \times y \times 2z \times 9.$$

$$20 \times 4 = x$$

$$x = 80$$

Hence the required number of days = 8

11) 15 person working 8 hours a day can complete a work in 21 days. How many days will 14 persons take to complete a work 1½ times as great if they work 6 hours a day?

Soln:

Let the required number of days be x.

Let the work done in the two cases y and $\frac{3}{2}$ respectively.

$M_1 = 15$. $D_2 = ?$ $W_2 = 1\frac{1}{2}y$

$T_1 = 8$ hrs $M_2 = 14$. $= \frac{2+1}{2}y$

$D_1 = 21$ days $T_2 = 6$ hrs $= \frac{3}{2}y$

$W_1 = y$. $W_2 = \frac{3}{2}y$.

Then the required number of days be x.

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1.$$

$$15 \times 8 \times 21 \times \frac{3}{2}y = 14 \times 6 \times x \times y.$$

$$5 \times 3 \times 3 = 2x.$$

$$x = 45.$$

Hence number of days is 45.

12) A contract is to be completed in 50 days and 105 men are set to work, each working 8 hrs a day. After 25 days, 2/5 of the work is finished. How many additional men be employed so that the work can be completed on time. Each man now working 9 hrs a day?

Soln:

Let the required number of additional men be x.

$M_1 = 105$ $\quad$ $M_2 = 105 + x$

$D_1 = 85$ days. $\quad$ $D_2 = 85$ days

$T_1 = 8$ hrs $\quad$ $T_2 = 9$ hrs

$W_1 = 2/5$. $\quad$ $W_2 = (1 - \frac{2}{5}) = 3/5$.

$$M_1 D_1 T_1 W_2 = M_2 D_2 T_2 W_1$$

$$105 \times 25 \times 8 \times \frac{3}{5} = (105 + x) \times 25 \times 9 \times \frac{2}{5}$$

$$140 = 105 + x$$

$$140 - 105 = x$$

$$x = 35.$$

Hence, additional number of men required be 35.

13) If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours a day, how much coal will be required for 8 engines each running 13 hours a day. It being given that 3 engines of former type consume as much as 4 engines of latter type?

Soln:

Let 3 engines of former type consume 1 unit in 1 hours.

Then 4 engines of latter type consume 1 unit in 1 hour.

-> 1 engine of former type consume 1/2 unit in 1 hour

-> 1 engine of latter type consumes 1/4 unit in 1 hrs.

Let the required consumption of coal be x units

$E_1 = 9$. $E_2 = 8$.

$T_1 = 8$ hrs. $T_2 = 13$ hrs.

$W_1 = 24$. $W_2 = ?$

Ratio of consumption 1/3 : 1/4

$$E_1 T_1 W_2 = E_2 T_2 W_1 \cdot \frac{2}{16}$$

$$\frac{9}{1} \times \frac{8}{1} \times x \times \frac{1}{8} = \frac{8}{1} \times 13 \times \frac{24}{1} \times \frac{1}{16}.$$

$$8x = 96.$$

$$x = 26.$$

Hence the required consume is 26.

4) A garrison of 3300 men had provisions for 32 days. When given at the rate of 850 gms. per head. At the end of 7 days. A reinforcement arrive and it was found that the provisions will last 17 days more. When given at the rate of 82.5 gms per head. What is the strength of the reinforcement?

Soln:

$M_1 = 3300$.

$D_1 = (32 - 7)$ days

$D_1 = 25$ days.

Ratio :- 825 : 850.

$w_1 = 1$.

$M_2 = ?$

$D_2 = 17$ days.

$w_2 = 1$.

Let the required men taking 825 gm each have provisions for 17 days be x.

$$M_1 D_1 w_2 = M_2 D_2 w_1$$

$$3300 \times 25 \times 850 = x \times 17 \times 825.$$

$$x = \frac{3300 \times 25 \times 850}{17 \times 825}$$

$$x = 5000.$$

Strength of reinforcement =

$$(5000 - 3800) = 1200$$

15) Two coal loading machines each working 12 hours per days for 8 days handle 9000 tones of coal with an efficiency of 90%. with 3 other coal loading machine of an efficiency of 80%. are said to handle 12000 tonnes of coal in 6 days. find how many hours per day each should work.

Soln:-

Let the number of working hours per day be x.

$M_1 = 2$

$D_1 = 8$.

$T_1 = 12$.

$W_1 = 9000$

efficiency $= 90\%$.

$M_2 = 3$.

$W_2 = 12000$

$D_2 = 6$

$T_2 = x$

Efficiency $= 80\%$

$$2 \times 8 \times 12 \times 12000 \times 90 = 3 \times 6 \times x \times 8 \times 9000$$

$$x = \frac{2 \times 8 \times 12 \times 12000 \times 90}{3 \times 6 \times x \times 8 \times 9000}$$

$$x = 16$$

Hence each machine should work for 16 hours per day.

Time and work.

Important Facts:

* If A can do a piece of work in n days then A's 1 days work = $1/n$.

* If A's 1 days work = $\frac{1}{n}$ then A can finish the work in n days.

* If A is thrice as good as workman as B, then.

⇒ Ratio of work done by A and B = 3:1

⇒ Ratio of times taken by A and B to finish a work = 1:3.

Problem:

1. If Roger can do a piece of work in 8 days and Antony can complete the same work in 5 days in how many days will both of them together complete it?

Soln:

Roger's 1 days's work = $\frac{1}{8}$.

Antony's 1 days's work = $\frac{1}{5}$.

(Roger + Antony)'s 1 day's work = $\left(\frac{1}{8}+\frac{1}{5}\right) = \frac{13}{40}$

∴ Both Roger and Antony will complete the work in $\frac{40}{13} = 3\frac{1}{13}$ days

2) A and B together can complete a piece of work in 15 days and B alone in 20 days. How many day can A alone complete the work?

Soln:

(A + B)'s 1 day's work $= \frac{1}{15}$.

B's 1 day's work $= \frac{1}{20}$.

A's 1 days's work + B's 1 days work $= \frac{1}{15}$.

A's (1 days) works + $\frac{1}{20} = \frac{1}{15}$.

A's 1 day's work $= \frac{1}{15} - \frac{1}{20}$.

$= \frac{1}{60}$.

Hence A alone can complete the work in 60 days.

3) a alone can complete a piece of work of ₹. 300 in 6 days; but by engaging an assistant. the work is completed in 4 days. Find the share to be received by the assistant

Soln:

Assistant 1 days' work $= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

∴ A's share : assistant share

= Ratio of their 1 days work.

$= \frac{1}{6} : \frac{1}{12}$

$= 2 : 1$

Hence assistant share $= \left(₹.300 \times \frac{1}{3}\right) =$

$= ₹.100$

4) A can do a work in 4 days, B in 5 days and C in 10 days. Find the time taken by A, B and C to do the work together.

Soln.

A's 1 day's work $= \frac{1}{4}$

B's 1 day's work $= \frac{1}{5}$

C's 1 day's work $= \frac{1}{10}$

(A+B+C)'s 1 day's work $= \left(\frac{1}{4} + \frac{1}{5} + \frac{1}{10}\right)$

$= \frac{11}{20}$.

Hence A, B and C together can do the work in $\frac{20}{11} = 1\frac{9}{11}$ days.

5) A and B undertake to do a piece of work for ₹.600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it 3 days. Find the share of each.

Soln.

C's 1 day's work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) = \frac{1}{24}$.

∴ A : B : C = Ratio of their 1 day's work

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{24}$

$= 4 : 3 : 1$.

A's share $= ₹\left(600 \times \frac{4}{8}\right) = ₹.300$.

B's share $= ₹\left(600 \times \frac{3}{8}\right) = ₹.225$)

C's share = ₹ $\left(600 \times \frac{1}{8}\right)$ = ₹75

b) A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it working together $8\frac{2}{5}$ hours a day?

Soln:

A can complete the work in $(7 \times 9) = 63$ hrs

B can complete the work in $(6 \times 7) = 42$ hr

∴ A's 1 hour's work = $\frac{1}{63}$.

B's 1 hour's work = $\frac{1}{42}$.

(A + B)'s 1 hours' work = $\frac{1}{63} + \frac{1}{42} = \frac{5}{126}$

Both will finish the work in $\left(\frac{126}{5}\right)$ hrs

∴ Number of days of $8\frac{2}{5}$ hrs each. = $\left(\frac{126}{5} \times \frac{5}{42}\right)$

= 3 days

7) Rahul takes twice as much time as Manish and thrice as much time as Sachin to complete a job. If working together they can complete the job in 4 days. Find the time taken by each of them separately to complete the work?

Soln:

Suppose Rahul takes x hours to complete job.

Then Manish takes $\frac{x}{2}$ hrs and Sachin takes $\frac{x}{3}$ hours to do the job.

Rahul's 1 hr's work $= \frac{1}{x}$.

Manish's 1 hr's work $= \frac{2}{x}$.

Sachin's 1 hr's work $= \frac{3}{x}$.

$\therefore 4\left(\frac{1}{x} + \frac{2}{x} + \frac{3}{x}\right) = 1$.

$4\left(\frac{6}{x}\right) = 1 \Rightarrow \frac{24}{x} = 1$.

$x = 24$

8. A and B can do a piece of work in 9 days, B and C can do it in 12 days. A and C can do it in 18 days. In how many days will A, B and C finish it working together and separately?

(A + B)'s 1 day's work $= \frac{1}{9}$.

(B+C)'s 1 day's work $= \frac{1}{12}$.

(A+C)'s 1 day's work $= \frac{1}{18}$.

Adding, we get.

2 (A+B+C)'s 1 day's work.

$= \left(\frac{1}{9} + \frac{1}{12} + \frac{1}{18}\right) = \frac{9}{36}$

$= \frac{1}{4}$.

∴ (A+B+C)'s 1 day's work $= \frac{1}{8}$

Thus, A, B and C together can finish the work in 8 days

Now, A's 1 days work =

[(A+B+C)'s 1 day's work − (B+C)'s 1 day's work]

$= \frac{1}{8} - \frac{1}{12}$

$= \frac{1}{24}$.

∴ A alone can finish the work in 24 days

Similarly B's 1 day's work.

= (A+B+C)'s 1 day work − (A+C)'s 1 days work.

$= \frac{1}{8} - \frac{1}{18} = \frac{5}{72}$

∴ B alone can finish the work in

$\frac{72}{5} = 14\frac{2}{5}$ days

and C's 1 day's work.

= [A+B+C)'s 1 days work] − (A+B)'s 1 days work.

$= \frac{1}{8} - \frac{1}{9} = \frac{1}{72}$.

∴ C alone can finish the days is 72 day

9) A is twice as good a workman as B and. together they finish a piece of work in 18 days In how many days will A alone finish the work?

Soln:

(A's 1days work) : (B's 1 days work) = 2 : 1

(A + B)'s 1day's work = $\frac{1}{8}$, Surde $\frac{1}{18}$ in the ratio 2 : 1.

∴ A's 1day's work = $\left(\frac{1}{18} \times \frac{2}{3}\right) = \frac{1}{27}$

Hence A alone can furnish the work in 27 days.

10) A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?

Soln:

Ratio of times taken by A and B = 160 : 100
= 8 : 5

Suppose B alone takes x days to do the job.

Then 8 : 5 :: 12 : x ⇒ 8x = 5 × 12 ⇒ x = $7\frac{1}{2}$ days

11) A can do a piece of work in 80 days. He works at it for 10days and then B alone finishes the remaining work in 42 days. In how much time will A and B, working together, finish the work?

Soln:

Work done by A in 10days = $\left(\frac{1}{80} \times \frac{1}{10}\right)$

= $\frac{1}{8}$.

Remaining work = $\left(1 - \frac{1}{8}\right) = \frac{7}{8}$.

Now $\frac{7}{8}$ work is done by B in 42 days

whole work will be done by B in $(42 \times \frac{8}{7}) = 48$ days

$\therefore$ A's 1 day's work $= \frac{1}{80}$

B's 1 day's work $= \frac{1}{48}$

$\therefore$ (A + B)'s 1 days work $= \frac{1}{80} + \frac{1}{48} = \frac{8}{240}$

$= \frac{1}{30}$

Hence, both will finish the work in 30 days

12) A can do a piece of work in 10 days and B in 20 days, they work together but 2 days before the completion the work. A leaves, In how many days was the work completed?

Soln:

B's 2 day's work $= \left(\frac{1}{20} \times 2\right) = \frac{1}{10}$.

Remaining work $= 1\left(1 - \frac{1}{10}\right) = \frac{9}{10}$

(A + B)'s 1 day's work $= \left(\frac{1}{10} + \frac{1}{20}\right) = \frac{6}{40} = \frac{3}{20}$

Now $\frac{3}{20}$ work is done by A and B in 1 day

$\therefore \frac{9}{10}$ work is done by A and B in $\left(\frac{20}{3} \times \frac{9}{10}\right)$

$= 6$ days

Hence total time taken $= (2+6)$ days

$= 8$ days.

18) A can complete a work in 10 days, B in 12 days and C in 15 days. All of them began the women work together, but A had to leave the work after 2 days of the start and B, 3 days before the completion of the work, How long did the work last?

Soln:

A, B and C work together for 2 days, C alone works for 3 days and the remaining work is done by B and C together.

Now (A+B+C)'s 2 day's work.

$= 2\left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right) = 2\left(\frac{15}{60}\right) = \frac{1}{2}$.

C's 3 days work $= \left(3 \times \frac{1}{15}\right) = \frac{1}{5}$.

Remaining work $= 1 - \left(\frac{1}{2} + \frac{1}{5}\right) = \frac{3}{10}$

But (B+C)'s 1 day's work $= \left(\frac{1}{12} + \frac{1}{15}\right)$

$= \frac{27}{180} = \frac{3}{20}$.

Now $\frac{3}{20}$ work is done by (B+C) in 1 days

$\therefore \frac{3}{10}$ work is done by (B+C) in $\left(\frac{20}{3} \times \frac{3}{10}\right) = 2$ day

Hence Total time taken $= (2+3+2)$ days

$= 7$ days

4). A and B can do a piece of work in 45 and 40 days respectively. They began the work together but A leaves after some days and B finished the remaining work in 23 days. After how many days did A leave?

Soln:

B's 23 days work = $\left(23 \times \frac{1}{40}\right)$

$= \frac{23}{40}$.

Remaining work = $\left(1 - \frac{23}{40}\right) = \frac{17}{40}$.

Now (A+B)'s 1 day's work = $\left(\frac{1}{45} + \frac{1}{40}\right) = \frac{17}{360}$.

Thus, $\frac{17}{360}$, work is done by (A+B) in 1 day.

$\frac{17}{40}$ work is done by (A+B)

in $\left(\frac{360}{17} \times \frac{17}{40}\right) = 9$ days.

Hence A left after 9 days

5) A and B working separately can do a piece of work in 9 and 12 days respectively alternately, A beginning, in how days the work will be completed?

Soln:

(A+B)'s 2 day's work = $\left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}$.

Work done in 5 pairs of days = $\left(5 \times \frac{7}{36}\right) = \frac{35}{36}$

Remaining work = $\left(1 - \frac{35}{36}\right) = \frac{1}{36}$

on 11th day, it is A's turn. $\frac{1}{9}$ work is done by him in 1 day.

$\frac{1}{36}$ work done by him in $(9 \times \frac{1}{36}) = \frac{1}{4}$ day.

$\therefore$ total time taken $= (10 + \frac{1}{4})$ days $= 10\frac{1}{4}$ days

16) A can do a piece of work in 180 days and B can do it in 150 days. They work together for 20 days. Then, B leaves and A alone continues the work. 12 days after that C joins A and the work is completed in 48 days more. In how many days can C do it. If he works alone?

Soln:

$= [(A+B)$'s 20 day's work$] + [A$'s (12 day's work)

$= 20\left(\frac{1}{120} + \frac{1}{150}\right) + 12\left(\frac{1}{120}\right)$

$= \frac{2}{5}$.

Remaining work $= (1 - \frac{2}{5}) = \frac{3}{5} = (A+C)$'s 48 day's work.

$\therefore (A+C)$'s 1 day's work $= \left(\frac{3}{5} \times \frac{1}{48}\right)$

$= \frac{1}{80}$

C's 1 day work $= (A+C)$'s 1 day's work $- A$'s 1

$= \frac{1}{80} - \frac{1}{120}$

$= \frac{1}{240}$

Hence C alone can finish the work in 240 days

17) A and B can do a piece of work in 12 days & B and C together can do it in 15 days. If A is twice as good a workman as C, find in what time B alone can do it.

Soln:

A's 1 day's work = C's 2 days work.

∴ (A+B)'s 1 day's work = (B's 1 days work) + C's 2 days work.

(B's 1 day work) + (C's 2 days work) = $\frac{1}{12}$ → (i).

(B's 1 day work) + (C's 1 day's work) = $\frac{1}{15}$.

Subtracting (ii) from (i).

we get: C's 1 days work = $\left(\frac{1}{12}-\frac{1}{15}\right) = \frac{1}{60}$

∴ B's 1 day's work = $\left(\frac{1}{15}-\frac{1}{60}\right) = \frac{3}{60} = \frac{1}{20}$

Hence B alone can finish the work in 20 days.

18) 45 men can complete a work in 16 days. Six days after they started working, 30 more men joined them. How many days will they now take to complete the remaining work?

Soln:

(45×16) men can complete the work in 1 day.

∴ 1 man's 1 day's work = $\frac{1}{720}$.

45 men's 1 day's work = $\frac{45}{720} = \frac{1}{16}$.

∴ 45 men's 6 days work = $\left(\frac{1}{16}\times 6\right) = \frac{3}{8}$.

Remaining work = $(1-\frac{3}{8})$

$= \frac{5}{8}$.

75 men's 1 day's work = $\frac{75}{720} = \frac{5}{48}$.

Now $\frac{5}{48}$ work is done by them in 1 day.

$\therefore \frac{5}{8}$ work is done by them in $(\frac{48}{5} \times \frac{5}{8}) = 6$ days.

19) 10 person begin to work together on a job, but after some days 4 person leave. As a result, the job which could have been completed in 40 days is completed in 80 days. How many days after the commencement of the work did the 4 person leave?

soln:

10 person can complete the work in 40 day

1 person 1 day's work = $\frac{1}{40 \times 10} = \frac{1}{400}$.

Suppose 4 person left after x days

$\frac{1}{400} \times 10 \times x + \frac{1}{400} \times 6 \times (50 - x) = 1$

$\frac{x}{40} + \frac{3(50-x)}{200} = 1$

$\frac{5x}{200} + \frac{150 - 3x}{200} = 1$

$\frac{5x + 150 - 3x}{200} = 1$

$\frac{2x + 150}{200} = 1 \Rightarrow \frac{2(x+75)}{200} = 1$

$\frac{x+75}{100} = 1 \Rightarrow x + 75 = 100 \Rightarrow x = 100 - 75$

$x = 25$.

Hence 4 person left 25 days after the commencement of the work.

20) 9 children can complete a piece of work in 360 days. 18 men can complete the same piece of work in 72 days. and 12 women can complete it in 162 days. In how many days can 4 men, 12 women and 10 children complete the piece of work?

Soln:

1 man's 1 days work = $\frac{1}{72 \times 18} = \frac{1}{1296}$

1 woman's 1 day's work = $\frac{1}{162 \times 12} = \frac{1}{1944}$

1 child's 1 day's work = $\frac{1}{360 \times 9} = \frac{1}{3240}$

(4 men + 12 women & + 10 children) 1 day's work.

$= \left(4 \times \frac{1}{1296}\right) + \left(12 \times \frac{1}{1944}\right) + \left(10 \times \frac{1}{3240}\right)$

$= \frac{1}{324} + \frac{1}{162} + \frac{1}{324}$

$= \frac{1}{81}$

Hence 4 men, 12 women and 10 children can complete the work in 81 days

21). 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Soln:

Let 1 man's 1 day's work = x.

1 Boy's 1 day's work = y.

Then $2x + 3y = \frac{1}{10}$, $3x + 2y = \frac{1}{8}$

Solving we get $x = \frac{7}{200}$, $y = \frac{1}{100}$

$\therefore$ (2 men + 1 boy) 1 day's work $= \left(2 \times \frac{7}{200}\right) + \left(1 \times \frac{1}{100}\right)$

$= \frac{16}{200}$

$= \frac{2}{25}$.

So 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days.

22) 3 men and 4 women can earn ₹. 3780 in 7 days. 11 men and 13 women can earn ₹. 15040 in 8 days. In what time will 7 men and 9 women earn ₹. 12400?

Soln.

1 man's 1 day's earning be ₹. x & 1 women's 1 day's earning be ₹ y.

$3x + 4y = \frac{3780}{7} = 540$.

$11x + 13y = \frac{15040}{8} = 1880$.

By solve $x = 100$, $y = 60$

7 men & 9 women $= (7 \times 100 + 9 \times 60) = ₹ 1240$

Hence required time $= \left(\frac{12400}{1240}\right)$ days $= 10$ days

Unit - 2.

Time and Distance.

Important Formula:

1) i) Speed = $\frac{\text{distance}}{\text{time}}$

ii) Time = $\frac{\text{distance}}{\text{speed}}$

iii) Distance = Speed × Time

2) x km/hr = $\left(x \times \frac{5}{18}\right)$ m/sec.

3) x m/sec = $\left(x \times \frac{18}{5}\right)$ km/hr.

4) If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$.
or $b : a$.

5) Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. then the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$ km/hr

6) Suppose two men are moving in the same direction at u m/s and v m/s respectively. where $u > v$, then their relative speed = $(u - v)$ m/s

7) Suppose the men are moving in opposite direction at u m/s and v m/s respectively. then relative speed is = $(u + v)$ m/s

8). If two person A and B start at the same time in opposite directions from two points and after passing each other they complete the journey's in a and b hours respectively. then.

A's speed : B's speed = $\sqrt{b} : \sqrt{a}$.

Book Back Exercise.

1). a train travels 82.6 km/hr. How many metres will it travel in 15 minutes?

Soln:

Distance travelled in 1 min = $\left(\frac{82.6}{60}\right)$ km.

∴ Distance travelled in 15 min = $\left[\frac{82.6}{60} \times 15\right]$ km

= 20.65 km.

= (20.65 × 1000) m.

= 20650 m.

2) How many minutes does Aditya take to cover a distance of 400 m. if he runs at a speed of 20 km/hr.?

Soln:

Aditya's speed = 20 km/hr.

= $\left(20 \times \frac{5}{18}\right)$ m/sec.

= $\frac{50}{9}$ m/sec.

Time taken to cover 400 m = $\left(400 \times \frac{9}{50}\right)$ sec.

= 72 sec.

$= 1\frac{12}{60}$ min

$= 1\frac{1}{5}$ min

3) A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

Soln:

$$\text{Speed} = \left[\frac{750}{150}\right] \text{m/sec}.$$

$$= 5 \text{ m/sec}.$$

$$= \left(5 \times \frac{18}{5}\right) \text{km/hr}.$$

$$= 18 \text{ km/hr}.$$

4) A man walked at a speed of 4 km/hr from point A to point B and came back from point B to A at the speed of 6 km/hr. What would be the ratio of the time taken by the man in walking from point A to point B to that from point B to A.

Soln:-

Ratio of speeds = 4 : 6

= 2 : 3

Ratio of time taken $= \frac{1}{2} : \frac{1}{3}$

= 3 : 2.

5) A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Soln:

Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the hare be y.

then $3x = 4y \Rightarrow x = \frac{4}{3}y$.

Multiply by 4 on both

$$4x = \frac{16}{3}y.$$

Ratio of speeds of dog and Hare.

= Ratio of distance covered by them in the same time.

$= 4x : 5y.$

$= \frac{16}{3}y : 5y$

$= \frac{16}{3} : 5 \Rightarrow 16 : 15.$

6) While covering a distance of 24 km, a man noticed that after walking 1 hr and 40 min. the distance covered by him was $\frac{5}{7}$ of the remaining distance. what was his speed in metres per second?

Soln:

Let the speed be x km / hr.

Then distance covered in 1 hr 40 min

(i.e) $1\frac{2}{3}$ hrs $= \frac{5x}{3}$ km.

Remaining distance $= \left(24 - \frac{5x}{3}\right)$ km

$$\frac{5x}{3} = \frac{5}{7}\left(24 - \frac{5x}{3}\right)$$

$$= \frac{5}{7}\left(\frac{72 - 5x}{3}\right) \quad \Rightarrow \quad \frac{5x}{3} = \frac{5(72 - 5x)}{3 \times 7}$$

$x = \frac{72-5x}{7}$ $\Rightarrow$ $7x = 72 - 5x$.

$7x + 5x = 72$

$12x = 72$

$x = 6$.

Hence speed $= 6$ km/hr.

$= 6 \times \frac{5}{18}$.

$= \frac{5}{3}$ m/sec.

$= 1\frac{2}{3}$ m/sec.

7) A trip to a destination is made in the following way: 900 km by train at an average speed of 60 km/hr, 3000 km by plane at an average speed of 500 km/hr. 400 km by boat at an average speed of 25 km/hr, 15 km by taxi at an average speed of 45 km/hr. What is the average speed for the entire journey?

Soln:

Total distance travelled $= (900 + 3000 + 400 + 15)$ km

$= 4315$ km.

Total time taken $= \left(\frac{900}{60} + \frac{3000}{500} + \frac{400}{25} + \frac{15}{45}\right)$ hr.

$= \left(15 + 6 + 16 + \frac{1}{3}\right)$ hr.

$= 37\frac{1}{3}$ hr.

$= \frac{112}{3}$ hr

$\therefore$ Average speed for the whole journey

$= \left(4315 \times \frac{3}{112}\right)$ km/hr

$= 115\left(\frac{35}{112}\right)$ km/hr

8) Bisola Peter can cover a certain distance in 1 hr. 24 min by covering two third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance

Soln:

Let the total distance be x km, then.

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5}$$

$$\frac{x}{6} + \frac{x}{15} = \frac{7}{5}$$

$$7x = 42$$

$$x = 6.$$

∴ Total distance = 6km

9) One-third of a certain journey was covered at the speed 20 km/hr. one-fourth at 30 km/hr and the at the speed of 50 km/hr. Find the average speed per hour for the whole journey

Soln:

Let the total distance be x km then

distance covered at 20 km/hr = $\frac{x}{3}$ km,

distance covered at 30 km/hr = $\frac{x}{4}$ km.

distance covered at 50 km/hr = $\left[x - \left(\frac{x}{3} + \frac{x}{4}\right)\right]$ km

$= \frac{5x}{12}$ km.

Total time taken = $\left[\frac{(x/3)}{20} + \frac{(x/4)}{30} + \frac{(5x/12)}{50}\right]$ hr

$= \frac{x}{60} + \frac{x}{120} + \frac{x}{120} = \frac{4x}{120}$ hr

$= \frac{x}{30}$ hr

∴ average speed for the whole journey = $(x \times \frac{30}{x})$

* 30 km/hr.

10) A man travelled from the village to the post office at the rate of 25 kmph and walked back to the 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post office from the village.

Soln:

$$\text{Average speed} = \left(\frac{2xy}{x+y}\right) \text{km/hr}.$$

$$= \left(\frac{2 \times 25 \times 4}{25+4}\right) \text{km/hr}.$$

$$= \frac{200}{29} \text{ km/hr}.$$

Distance travelled in 5 hrs 48 minutes (i.e) $5\frac{4}{5}$ hrs

$$= \left(\frac{200}{29} \times \frac{29}{5}\right) \text{km}$$

$$= 40 \text{ km}.$$

∴ Distance of the post office from the village $= \left(\frac{40}{2}\right) = 20$ km.

11) An aeroplane flies along the four sides of a square at the speeds of 100, 200, 300 and 400 km/hr. Find the average speed of the plane around the field.

Soln:

Let each side of the square be x km and let the average speed of the plane around the

field be y km.

Then,

$$\frac{x}{100} + \frac{x}{200} + \frac{x}{300} + \frac{x}{400} = \frac{4x}{y}.$$

$$\frac{12x + 6x + 4x + 3x}{1200} = \frac{4x}{y}.$$

$$\frac{25x}{1200} = \frac{4x}{y}.$$

$$y = \frac{1200 \times 4}{25}.$$

$$y = 192.$$

Average speed = 192 km/hr

12) A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train. Find the speed of the two trains.

Soln:

Let the speed of the fast train be x km/h

The speed of the slow train = $(x-10)$ km/h

$$\frac{600}{(x-10)} - \frac{600}{x} = 3$$

$$600x - 600(x-10) = 3x(x-10)$$

$$600x - 600x + 6000 = 3x^2 - 30x.$$

$$3x^2 - 30x - 6000 = 0.$$

$$\div \text{ by } 3$$

$$x^2 - 10x - 2000 = 0$$

$(x-50)(x+40) = 0$.

$x = 50$.

Hence speed of fast train = 50 km/hr.

and speed of slow train = 40 km/hr

13) By walking at $3/4$ of his usual speed, a man reaches his office 20 min later than his usual. Find the usual time taken by him to reach the office.

Soln:

New speed = $\frac{3}{4}$ of usual speed.

New time taken = $\frac{4}{3}$ of usual time.

So ($\frac{4}{3}$ of the usual time) - usual time = 20 min

$\frac{1}{3}$ of the usual time = 20 min

usual time = 20×3

= 60 min

= 1 hr.

14) A person reaches his destination 40 minutes late if his speed is 3 km/hr and reaches 80 minutes before time if his speed is 4 km/hr. Find the distance of his destination from his starting point

Soln:

Let the required distance be x km.

Difference in the times taken at two speeds

= 70 min = $\frac{7}{6}$ hr

$\frac{x}{3} - \frac{x}{4} = \frac{7}{6} \Rightarrow 4x - 3x = 14.$

$x = 14$

Hence the required distance is 14 km.

15) A carriage driving in a fog passed a man who was walking at the rate of 3 kmph. In the same direction. He could see the carriage for 4 minutes and it was visible to him upto a distance of 100 m. What was the speed of the carriage?

Soln:

Let the speed of the carriage be x kmph.

Then relative speed. $= (x-3)$ kmph.

Distance covered in 4 min i.e. $= \frac{1}{15}$ hr. at relative speed $= 100\text{m} = \frac{1}{10}$ km.

$\therefore (x-3) = \frac{1}{10} \times 15 \Rightarrow \frac{3}{2}.$

$x = 3 + \frac{3}{2} = \frac{9}{2}.$

$x = 4\frac{1}{2}.$

Hence Speed of the carriage $= 4\frac{1}{2}$ kmph

16) A train after travelling 150 km meets an accident and then proceeds at $\frac{3}{5}$ of its former speed and arrives at its destination 6 hrs late. Had the accident occured 80 km

further. it would have reached the destination 4 hours late. What is the total distance travelled by the train?

Soln:-

Let the original speed of the train be x km/h

$$\frac{360}{\left(\frac{5}{6}x\right)} - \frac{360}{x} = 4.$$

$$\frac{600}{x} - \frac{360}{x} = 4.$$

$$\frac{240}{x} = 4 \Rightarrow x = 60$$

Let the total distance travelled by the train be y km.

Then, $\left[\frac{150}{60} + \frac{(y-150)}{\left(\frac{5}{6}\times 60\right)}\right] - \frac{y}{60} = 8.$

$$\frac{5}{2} + \frac{y-150}{36} - \frac{y}{60} = 8.$$

$$2x = 1740$$

$$x = 870.$$

Hence required distance = 870 km

17) A ship 77 km from the shore, springs a leak which admits $2\frac{1}{2}$ tonnes of water in $5\frac{1}{2}$ minutes. 92 tonnes of water would sink it. But the pumps can throw out 12 tonnes of water per hour. Find the average rate of sailing so that the ship may just reach the shore as it begins to sink.

Soln:

Quantity of water let in by the leak in 1 min.

$= \frac{(9+\frac{1}{11})}{(5\frac{1}{2})}$ tonnes

$= \left(\frac{9}{11} \times \frac{2}{11}\right)$ tonnes.

$= \frac{9}{99}$ tonnes

Quantity of water thrown by the pumps in 1 min $= \left(\frac{12}{60}\right)$ tonnes $= \frac{1}{5}$ tonnes

Net quantity of water filled in the ship in 1 min $= \left(\frac{9}{22} - \frac{1}{5}\right)$ tonnes

$= \frac{23}{110}$ tonnes.

$\frac{23}{110}$ tonnes water is filled in 1 min.

92 tonnes water is filled in $\left(\frac{110}{23} \times 92\right)$ min

$= 440$ min

$= \frac{22}{3}$ hrs

Hence required speed $= \frac{77}{\frac{22}{3}}$ km/hr

$= \frac{21}{2} = 10.5$ km/hr

15) Excluding the stoppages the speed of the bus is 64 km/hr and including the stoppages, the speed of the bus is 48 km/hr Find how many minutes does the bus stop per hr?

Soln:

Due to stoppage, the bus covers = 64 − 48

= 16 km less per hr

Time taken to cover 16 km = $\left(\frac{16}{64} \times 60\right)$ min

= 15 min

Hence stoppage of time per hr = 15 min

19) An aeroplane started 30 minutes later than the scheduled time from a place 1500 km away from its destination. At the scheduled time the pilot had to increase the speed by 250 km/hr. What was the speed of the aeroplane per hour during the journey?

Soln:

Let the original speed of the aeroplane be x km/hr.

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$3000(x+250) - 3000x = x(x+250)$$

$$x^2 + 250x - 750000 = 0$$

$$x^2 + 1000x - 750x - 750000 = 0$$

$$x(x+1000) - 750(x+1000) = 0$$

$$(x-750)(x+1000) = 0$$

$$x = 750$$

Hence the speed of the aeroplane during the journey = 750 + 250 km/hr

= 1000 km/hr

80) Two boys A and B start at the same time to ride from Delhi to Meerut. 60 km away. A travels 4 km an hour slower than B reaches meerut and at once turns back meeting A 12 km from Meerut. Find A's speed

Soln:

Let A's speed = x km/hr.

B's speed = (x + 4) km/hr.

Clearly time taken by B to cover (60 + 12)

(i.e) 72 km = time taken by A to cover (60 − 12) i.e) = 48 km

$$\frac{72}{x+4} = \frac{48}{x}$$

$$72x = 48x + 192$$

$$24x = 192$$

$$x = 8.$$

81) A man covers a certain distance on a toy train. Had the train moved 4 km/hr. faster it would have taken 30 min less. If it moved 2 km/hr slower, it would have taken 40 min more. Find the distance.

20

Soln:

Let the distance be x km

and initial speed be y km/hr.

$$\frac{x}{y} - \frac{x}{y+4} = \frac{30}{60} = \frac{1}{2}.$$

$$\frac{2x}{y(y+4)} = 8. \Rightarrow 8x = y^2 + 4y.$$

$$x = \frac{y^2+4y}{8} \rightarrow (i).$$

$$\frac{x}{y-2} - \frac{x}{y} = \frac{20}{60} = \frac{1}{3}$$

$$\frac{2x}{y(y-2)} = \frac{1}{3}$$

$$6x = y^2 - 2y.$$

$$x = \frac{y^2-2y}{6} \rightarrow (2)$$

From (i) (ii).

$$\frac{y^2+4y}{8} = \frac{y^2-2y}{6}$$

$$6y^2 + 24y = 8y^2 - 16y$$

$$2y^2 = 40y$$

$$y = 20$$

Put $y = 20$.

$$x = 60.$$

Hence distance = 60 km

20) A and B are two stations 390 km apart. A train starts from A to 10.a.m and travels towards B at 65 km. another train start from B at 11 a.m. and travels towards A

at 85 kmph. At what time do they meet?

Soln:

They meet x hours after 10 a.m then

(Distance moved by first in x hrs)

+ (Distance moved by second in $(x-1)$ hrs]

$= 390$

$65x + 85(x-1) = 390$.

$100x = 425$.

$x = 4\frac{1}{4}$.

So they meet 4 hrs. 15 min after 10 a.m

i.e) at 8.15 pm.

23) A good train leaves a station at a certain time and at a fixed speed. After 6 hours an express train leaves the same station and moves in the same direction at a uniform speed of 90 km ph. This train catches up the good train in 4 hours. Find the speed of the good train.

Soln:

Let the speed of the goods train be x kmph

Distance covered by goods train in 10 hours.

= Distance covered by express train in 4h

$10x = 4 \times 90$ or

$x = 36$.

So speed of good train = 36 kmph.

EX. 24. A thief is spotted by a police-man from a distance of 100 metres. When the policeman 10 km/hr, how far

Soln:

Relative speed of the policeman } = (10 − 8) km/hr
= 2 km/hr.

Time taken by policeman to cover 100m } = $\left(\frac{100}{1000} \times \frac{1}{2}\right)$ hr.
= $\frac{1}{20}$ hr.

In $\frac{1}{20}$ hrs, the theif covers a distance of

$(8 \times \frac{1}{20})$ km = $\frac{2}{5}$ km = 400m.

25) Two places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction, they meet each other in 8 hrs. If they move in opposite directions towards each other, they meet in 1 hr 20 minutes. Determine the speeds of the car.

Soln:

Let their speeds x kmph and y kmph respectively

Then $\frac{80}{x-y} = 8 \Rightarrow x - y = 10$

$$\frac{80}{x+y} = 1\frac{1}{3} \Rightarrow \frac{80}{x+y} = \frac{4}{3}$$

$$60 = x+y \rightarrow (2)$$

adding 1 and 2 ..

$$2x = 70 \Rightarrow x = 35.$$

Put $x = 35$ in eqn (1)

$$y = 25$$

Hence the speed of two cars be 35 kmph and 25 kmph.

Q6) A man takes 6 hrs 30 min in going by a cycle and coming back by scooter. He would have lost 6 hours 10 min by going on cycle both ways. How long would it take him to go by scooter both ways?

Soln:

Let the distance be x km, then.

Time taken to cover x km by the cycle + time taken to cover x by scooter } = 6 hr 30

⇒ Time taken to cover 2x km by cycle + time taken to cover 2x km by scooter } = 13 hr

But time taken to cover 2x km by cycle } = 8 hr 40 min

Time taken to cover 1 km by Arcelo

= 13 hrs - 8 hrs 40 min

= 4 hr 20 min

Hence required time = 4 hr 30 min.

27) Sneha is picked up by her father by car from college everyday. The college gets over at 4 P.M daily one day. the college got over an hour earlier than usual. Sneha started walking towards her house. Her father, unaware of this fact, leaves his house as usual, meets his daughter on the way, picks her up and they reach the house 15 minutes earlier than usual. What is the ratio of the father's driving speed to Sneha's walking speed?

Soln:

Since 15 minutes are saved, it means that Sneha's father drives from the meeting point to the college and back to the meeting point in 15 min.

(i.e) he can drive from the meeting point to the college in $\left(\frac{15}{2}\right) = 7.5$ min.

But he reaches the college 4 P.M. so Sneha and her father meet on the way at $3:52\frac{1}{2}$ pm. $= \frac{\text{Sneha}}{\text{Father}} = \frac{7}{7}$

$= 1:1$

Unit - III.

Problems On Trains

Important Sums :

1) a km/hr = $(a \times \frac{5}{18})$ m/s

2). a m/s = $(a \times \frac{18}{5})$ km/hr.

3). Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.

4). Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover (l+b) metres.

5). Suppose two trains or two bodies are moving in the same direction at u m/s and v m/s where u > v then relative speed = (u-v) m/s.

6) Suppose two trains or two bodies are moving in opposite direction at u m/s and v m/s. then relative speed = (u+v) m/s

7). If two trains of length a metres and b metres are moving in opposite direction at u m/s and v m/s. then time taken by the train

to cross each other = $\frac{a+b}{u+v}$ sec.

8). If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s then the time taken by the faster train to cross the slower train = $\left(\frac{a+b}{u-v}\right)$ sec.

9). If two train (or bodies) start the same time from point A and B towards each other and after crossing they take a and b sec in reaching B and A respectively. then (A's speed) : (B's speed) = $(\sqrt{b} : \sqrt{a})$.

Problems:-

1) A 100m long train is running at the speed of 30 km/hr. Find the time taken by it pass a man standing near the railway line.

Soln.

speed of the train = $\left(30 \times \frac{5}{18}\right)$ m/sec

$= \frac{25}{3}$ m/sec.

Distance moved in passing the standing man = 100 m.

Required time taken = $\frac{100}{\frac{25}{3}} = \left(100 \times \frac{3}{25}\right)$

= 12 sec.

2) A train is moving at a speed of 132 km/hr. If the length of the train is 110 m. how long will it take to cross a railway platform 165 m long?

Soln:

Speed of train $= \left(132 \times \frac{5}{18}\right)$ m/sec.

$= \frac{110}{3}$ m/sec.

Distance covered in passing the platform $= (110 + 165)$ m $= 275$ m.

$\therefore$ Time taken $= \left(275 \times \frac{3}{110}\right)$ sec $= \frac{15}{2}$ sec

$= 7\frac{1}{2}$ sec.

3) A 160 m long train cross a 160 m long platform in 16 seconds. Find the speed of the train.

Soln;

Distance covered in passing the platform $= (160 + 160)$ m

$= 320$ m.

Speed of train $= \frac{320}{16}$ m/sec.

$= 20$ m/sec

$= \left(20 \times \frac{18}{5}\right)$ km/hr.

$= 72$ km/hr.

4) A person standing on a railway platform noticed that a train took 21 seconds to completely pass through the platform which was 84 m long and it took 9 seconds in passing him. Find the speed of the train.

Soln:

Let the length of the train be x metre.

Then the train covers x metre in 9 seconds and (x+84) metres in 21 seconds

So length of the train = 63 m.

Speed of the train $= \frac{63}{9}$ m/sec

$= 7$ m/sec.

$= 7 \times \frac{18}{5}$ km/hr

$= \frac{126}{5}$ km/hr.

$= 25.2$ km/hr

5) A train travelling with constant speed cross a 90m long platform in 10 seconds. And a 120 m long platform in 15 seconds Find the length of the train and its speed.

Soln:

Let the length of the train be x metres

Then $\frac{x+90}{10} = \frac{x+120}{15} \Rightarrow$

$15(x+90) = 10(x+120)$

$15x + 1350 = 12x + 1440$.

$3x = 90$

$x = 30$

Speed of the train $= \frac{30+90}{12}$ m/sec

$= 10$ m/sec.

$= \left(10 \times \frac{18}{5}\right)$ km/hr

$= 36$ km/hr.

Hence length of train $= 30$m

Speed of the train $= 36$ km/hr

6) A 150-m long train is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Soln:

Speed of the train relative to man $= 68 - 8$ kmph

$= 60$ kmph.

$= \left(60 \times \frac{5}{18}\right)$ m/sec.

$= \frac{50}{3}$ m/sec.

Time taken by the train to pass the man

= Time taken by it to cover 150m at

$\frac{50}{3}$ m/sec.

$\Rightarrow \left(150 \times \frac{3}{50}\right)$ sec $= 9$ sec.

7) A 220 m long train is running with a speed of 59 kmph. In what time will it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

Soln:

Speed of the train relative to man

$= (59 + 7)$ kmph.

$= \left(66 \times \frac{5}{18}\right)$ m/sec.

$= \frac{55}{3}$ m/sec.

Time taken by the train to cross the man

= time taken by it to cover 220 m at $\frac{55}{3}$ m/sec.

$= \left(220 \times \frac{3}{55}\right)$ sec $= 12$ sec.

8) Two trains 140 metre and 170 metre in length are running towards each other on parallel line one at the rate of 60 kmph and another at 48 kmph. How much time will they take to cross each other?

Soln:

Relative speed of two trains $= 60 + 48$.

$= 108$ kmph.

$= \left(108 \times \frac{5}{18}\right)$ m/sec

$= 30$ m/sec.

Time taken by the train to pass each.

pass each other.

= Time taken to covers (240 + 260) m

At 80 m/sec.

$= \frac{510}{30}$ sec

$= 17$ sec

9) A 300 m long train passed a man walking along the line in the same direction at the rate of 3 km/hr in 33 seconds. Find the speed of the train in km/hr?

Soln:

Speed of the train relative to man $= \frac{300}{33}$ m/s $= \frac{100}{11}$ m/sec.

$= \left[\frac{100}{11} \times \frac{18}{5}\right]$ km/hr.

$= \frac{360}{11}$ km/hr.

Let the speed of the train be x km/hr.

Then relative speed $= x - 3$ km/hr.

$x - 3 = \frac{360}{11}$.

$x = \frac{360}{11} + 3 \Rightarrow x = \frac{393}{11}$.

$x = 35\frac{8}{11}$.

Hence speed of train $= 35\frac{8}{11}$ km/hr

10) Two trains 100 metres and 120 meters long are running the same direction with speeds of 70 km/hr and 54 km/hr. In how much time the first train cross the second?

Soln:

Relative speed of the train = (70 − 54) km/hr

= 18 km/hr

$= \left(18 \times \frac{5}{18}\right)$ m/sec

= 5 m/sec.

time taken by the train to pass each other

= Time taken to cover (100 + 120) m at 5 m/sec

$= \frac{220}{5}$ m/1 sec = 44 sec.

11) A 100 m long train takes $7\frac{1}{5}$ seconds to cross a man walking at a rate of 5 km/hr in the direction opposite to that of the train, what is the speed of the train?

Soln:

Let the speed of the train be x km/hr

Speed of the train relative to man = (x + 5) km/hr.

$= (x+5) \times \frac{5}{18}$.

$$\frac{100}{(x+5) \times \frac{5}{18}} = \frac{36}{5}$$

$$10x + 50 = 500 \Rightarrow 10x = 450$$

$$x = 45$$

Hence speed of the train = 45 km/hr.

12) A train 100 m long travelling at 60 km/hr passes another train twice as fast as the train and travelling in opposite direction in 10 seconds. Find the length of the second train

Soln:

Relative speed = (60 + 120) km/hr.

$= \left(180 \times \frac{5}{18}\right)$ m/sec.

= 50 m/sec.

Let the length of the second train be x metres.

Then, $\frac{x+100}{10} = 50$.

$x + 100 = 500$

$x = 400$

Hence length of second train = 400m

13) A train running at 54 kmph takes 20 sec to pass a platform. Next it takes 12 seconds to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.

Soln:

Let the length of train be x metres and the length of platform be y metres.

Speed of the train relative to man

$= (54 - 6)$ kmph.

$= 48$ kmph.

$= \left(48 \times \frac{5}{18}\right)$ m/sec $= \frac{40}{3}$ m/sec.

In passing a man, the train covers its own length with relative speed.

$\therefore$ length of train = (relative speed × time)

$= \left(\frac{40}{3} \times 12\right)$ m.

$= 160$ m

Also speed of the train $= \left(54 \times \frac{5}{18}\right)$ m/sec

$= 15$ m/sec.

$\frac{x+y}{15} = 20 \Rightarrow x + y = 300$.

Here $x = 160$ m.

$160 + y = 300$

$y = 140$ m

4). A moving train 66 metres long, overtakes another train 88 metres long, moving in the same direction, in 0.168 minutes. If the second train is moving at 30 km per hour. at what speed is the first train moving?

Soln:

Let the speed of the first train be x km/hr.

Then sum of length of trains $= (66+88)$ m
$= 154$ m

Relative speed of two trains $= (x-30)$ kmph.
$= \left[(x-30) \times \frac{5}{18}\right]$ m/sec.

$$\frac{154}{(x-30)\times\frac{5}{18}} = 6.168 \times 60.$$

$$x = 30 + 55 = 85$$

Hence speed of the first train $= 85$ kmph

15) A man sitting in a train which is travelling at 50 kmph observes that a good train travelling in opposite direction takes 9 seconds to pass him. If the good train is 280m find its speed.

Soln:

Relative speed $= \frac{280}{9}$ m/sec.

$= \left[\frac{280}{9} \times \frac{18}{5}\right]$ kmph

$= 112$ kmph

Speed of good train $= (112 - 50)$ kmph

$= 62$ kmph.

Unit - 4

Boats and Streams

Important Facts :-

1). In water, the direction along the stream is called downstream. and the direction against the stream is called upstream.

2) If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr then,

* Speed downstream = (u+v) km/hr
* Speed upstream = (u-v) km/hr

3) If the speed downstream is a km/hr and the speed upstream is b km/hr then

* Speed in still water = $\frac{1}{2}(a+b)$ km/hr
* Rate of stream = $\frac{1}{2}(a-b)$ km/hr.

4). Suppose a man can swim in still water at the rate of u km/hr. The speed of current / stream is v km/hr. and the man wishes to cross the stream (of width x mts) straight along its width then time taken cross the river is the same as time taken to swim x mts at u km/hr

Note: This is because the stream speeds the man such that both the distance and the effective velocity increase and the time taken to cross the river remains unaffected

5) A man can swim directly across a width x km in t hours when there is no current and in 't' hours when there is a current. Then the rate of the current is

$$x\sqrt{\frac{1}{t^2}-\frac{1}{t'^2}} \text{ km/hr}$$

Problems:

1) A speed of a boat when travelling downstream is 32 km/hr., when travelling upstream it is 28 km/hr. What is the speed of the boat in still water and the speed of the stream?

Soln:

$$\text{Speed of boat in still water} = \frac{1}{2}(32+28) \text{ km} = 30 \text{ km/hr}$$

$$\text{Speed of stream} = \frac{1}{2}(32-28) \text{ km/hr} = 2 \text{ km/hr}$$

2) A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 6 hours 30 min. cover a distance of 5 km upstream. Find the speed of the river current in km/hr.

Soln:

Rate downstream $= \left(\frac{15}{3\frac{3}{4}}\right)$ km/hr.

$= \left[15 \times \frac{4}{15}\right]$ km/hr.

$= 4$ km/hr

Rate upstream $= \left(\frac{5}{6\frac{1}{2}}\right)$ km/hr.

$= \left(5 \times \frac{2}{5}\right)$ km/hr.

$= 2$ km/hr.

$\therefore$ Speed of current $= \frac{1}{2}(4-2)$ km/hr.

$= 1$ km/hr.

3) The speed of a motor boat is that of the current of water 36 : 5. The boat goes along with the current 5 hours 10 minutes. How much time will it take to come back?

Soln:

Let the speed of the motor boat and that of the current be $36x$ km/hr and $5x$ km/hr respectively.

Then speed downstream $= (36x + 5x)$ km/hr.

Speed upstream $= (36x - 5x)$ km/hr.

$= 31x$ km/hr

Let the distance be d km.

Then $\frac{d}{41x} = 5\frac{10}{60} = 5\frac{1}{6} = \frac{31}{6}$.

$d = \left(\frac{31 \times 41}{6}\right)x$.

$d = \frac{1271x}{6}$

Time taken while coming back $= \frac{d}{31x}$.

$= \frac{\frac{1271x}{6}}{31x}$.

$= \frac{1271x^2}{6} \times \frac{1}{31x}$

$= \frac{41}{6}$ hrs $= 6\frac{5}{6}$ hrs

$= 6$ hrs 50 min.

4) A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. Find the rate of stream.

Soln:

Let man's rate upstream be x km/hr

Then his rate downstream $= 2x$ km/hr

Rate in still water $= \frac{1}{2}(x + 2x)$ km/hr

$= \frac{3x}{2}$ km/hr.

$\frac{8x}{2} = 8^2 \Rightarrow x = 4$

Rate upstream = 4 km/hr.

Rate downstream = 8 km/hr.

Hence rate of stream = $\frac{1}{2}(8-4)$ km/hr.

= 2 km/hr.

5) There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 10 km/hr. While the other sails on a boat at a speed of 10 km/hr. If the river flows at the speed of 4 km/hr which of the two friend will return to place A first?

Soln:

Clearly the cyclist moves both ways at a speed of 10 km/hr.

So average speed of the cyclist = 12 km/hr

The boat sailor moves downstream a (10+4) i.e 14 km/hr and upstream is (10-4) at 6 km/hr

So average speed of the boat sailor

$= \left(\frac{2 \times 14 \times 6}{14+6}\right)$ km/hr $= \frac{42}{5}$ km/hr

$= 8.\dot{3}$ km/hr.

Since the average speed of the cyclist is greater, he will return to A first.

6) A man can row $7\frac{1}{2}$ kmph in still water. If in a river running at 1.5 km an hour it takes him 50 minutes to row a place and back, how far off is the place?

Soln:

Speed downstream $= (7.5 + 1.5)$ kmph $= 9$ kmph

Speed upstream $= (7.5 - 1.5)$ kmph $= 6$ kmph

Let the required distance be x km. Then

$$\frac{x}{9} + \frac{x}{6} = \frac{50}{60}$$

$$3x + 2x = \frac{5}{6} \times 18$$

$$5x = 15.$$

$$x = 3.$$

Hence the required distance is 3 km.

7) A boat goes 8 km upstream and then return. Total time taken is 4 hrs 16 minutes. If the velocity of current is 1 km/hr. Find the actual velocity of the boat?

Soln:

Let the actual velocity of the boat be x km/hr. Then,

speed downstream = $(x+1)$ km/hr ;

speed upstream = $(x-1)$ km/hr.

$$\frac{8}{x+1} + \frac{8}{x-1} = 1\frac{16}{60}.$$

$$\frac{8(x-1)+8(x+1)}{x^2-1} = \frac{256}{60}$$

$$\frac{8[(x-1)+(x+1)]}{x^2-1} = \frac{256}{60}$$

$$\frac{x-1+x+1}{x^2-1} = \frac{8}{15}.$$

$$\frac{8x}{x^2-1} = \frac{8}{15}.$$

$$30x = 8x^2 - 8.$$

$$8x^2 - 30x - 8 = 0.$$

$$4x^2 - 15x - 4 = 0.$$

$$(x-4)(4x+1) = 0$$

$$x = 4.$$

Hence actual velocity of the boat = 4 km/hr.

8) A boatman rows to a place 45 km distant and back in 20 hours. He finds that he can row 18 km with upstream in the same time as 4 km against the stream. Find the speed of the stream.

Soln:

Suppose he travels 18 km downstream in x hours. Then.

Speed downstream $= \frac{12}{x}$ km/hr

Speed upstream $= \frac{4}{x}$ km/hr.

$$\frac{45}{12/x} + \frac{45}{4/x} = 20.$$

$$\frac{x}{12} + \frac{x}{4} = \frac{20}{45}.$$

$$\frac{4x + 12x}{48} = \frac{4}{9}$$

$$\frac{16x}{48} = \frac{5}{9}$$

$$x = \frac{4}{3}$$

So. Speed downstream $= \left(12 \times \frac{3}{4}\right)$ km/hr.

$= 9$ km/hr.

Speed upstream $= \left(4 \times \frac{3}{4}\right)$ km/hr

$= 3$ km/hr.

∴ Speed of the stream $= \frac{1}{2}(9 - 3)$ km/hr.

$= 3$ km/hr.

9) A man can row 40 km upstream and 55 km downstream in 13 hours. Also he can row 30 km upstream 44 km downstream in 10 hours. Find the speed of the man in still water and the speed of the current.

Soln:

Let rate upstream = x km/hrs.

rate downstream = y km/hrs

Then $\frac{110}{x} + \frac{55}{y} = 13 \rightarrow (1)$.

$\frac{30}{x} + \frac{44}{y} = 10 \rightarrow (2)$

Multiply (ii) by 4 and (i) by 3 and subtracting we get $\frac{11}{y} = 1 \Rightarrow y = 11$.

Substituting $y = 11$ in we get $x = 5$.

Rate in still water $= \frac{1}{2}(11+5) = 8$ kmph.

Rate of current $= \frac{1}{2}(11-5)$ kmph.

$= 3$ kmph.

Unit - 5.

Alligation or Mixture

Important Facts :-

1). **Alligation :-**

It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

2). **Mean Price :-**

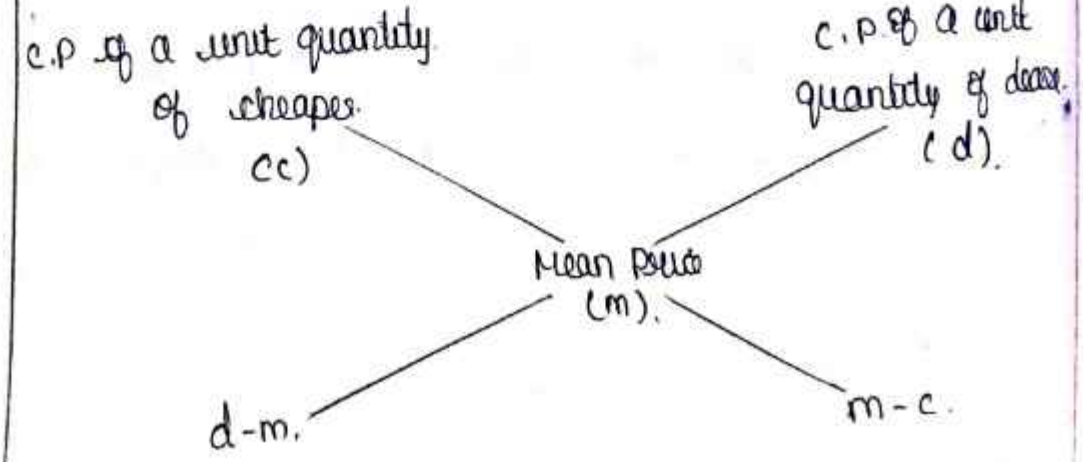
The cost price of a unit quantity of the mixture is called the mean price

3) **Rate of Alligation :-**

If two ingredients are mixed then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{C.P of dearer} - \text{Mean Price}}{\text{Mean Price} - \text{C.P of cheaper}}$$

We present as under :



cheaper quantity : dearer quantity = $(d-m) : m-c$.

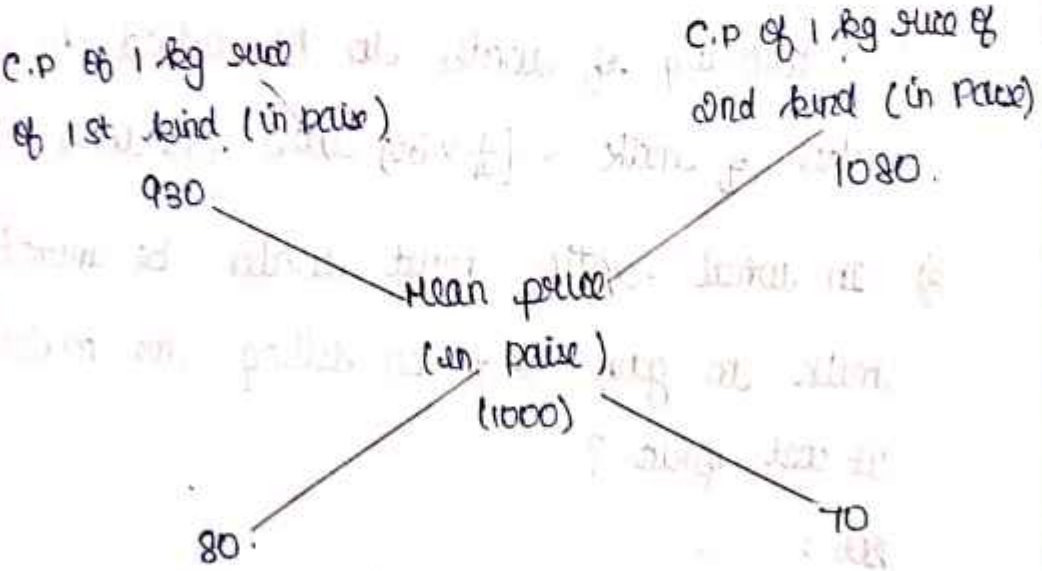
Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations the quantity of pure liquid = $\left[x\left(1-\frac{y}{x}\right)^n\right]$ units.

Problems:-

1) In what ratio must rice at ₹. 9.30 per kg be mixed with rice at ₹. 10.80 per kg so that the mixture be worth ₹. 10 per kg ?

Soln:-

By the rule of alligation, we have.



Required Ratio :- = 80 : 70

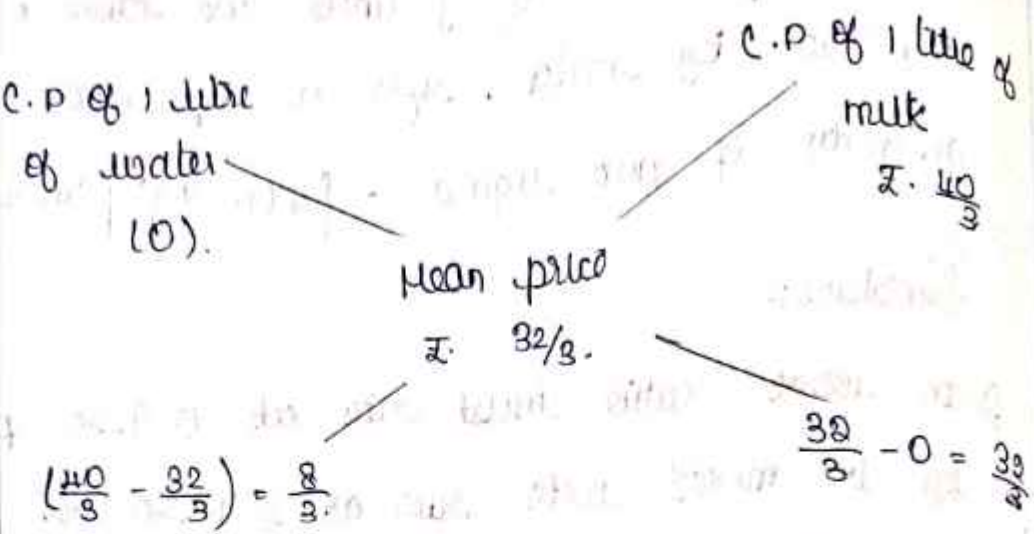
= 8 : 7.

2) How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for ₹. 00 so as to have a mixture worth ₹. $10\frac{2}{3}$ a litre ?

Soln:

C.P of 1 litre of milk = ₹. $(60 \times \frac{2}{3})$

= ₹. $\frac{40}{3}$



Ratio of water and milk

= $\frac{8}{3} : \frac{32}{3}$

= 1 : 4.

∴ Quantity of water to be added to 60 litres of milk = $(\frac{1}{4} \times 60)$ litres = 15 litres.

3) In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price?

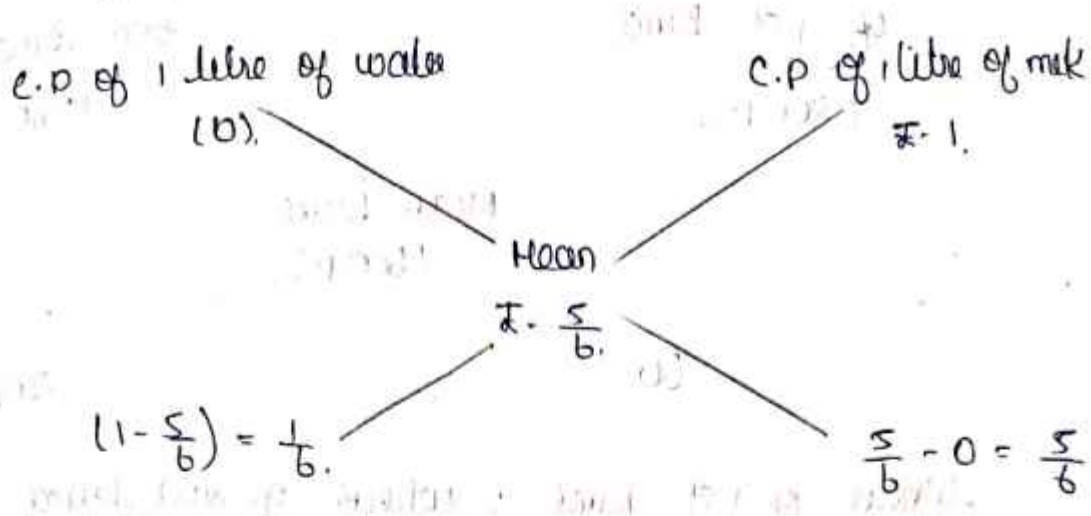
Soln:

C.P of 1 litre of mixture

= ₹. $(\frac{100}{120} \times 1)$

= ₹. $\frac{5}{6}$.

By the rule of allegation, we have.



Ratio of water and milk $= \frac{1}{6} : \frac{5}{6} = 1 : 5$

4) How long many kgs of wheat costing ₹. 8 per kg must be mixed with 36 kg of rice costing ₹. 5.40 per kg so to ₹. 20 % gain may be obtained by selling the mixture at ₹. 7.20 per kg ?

Soln:

S.P of 1 kg mixture = ₹. 7.20

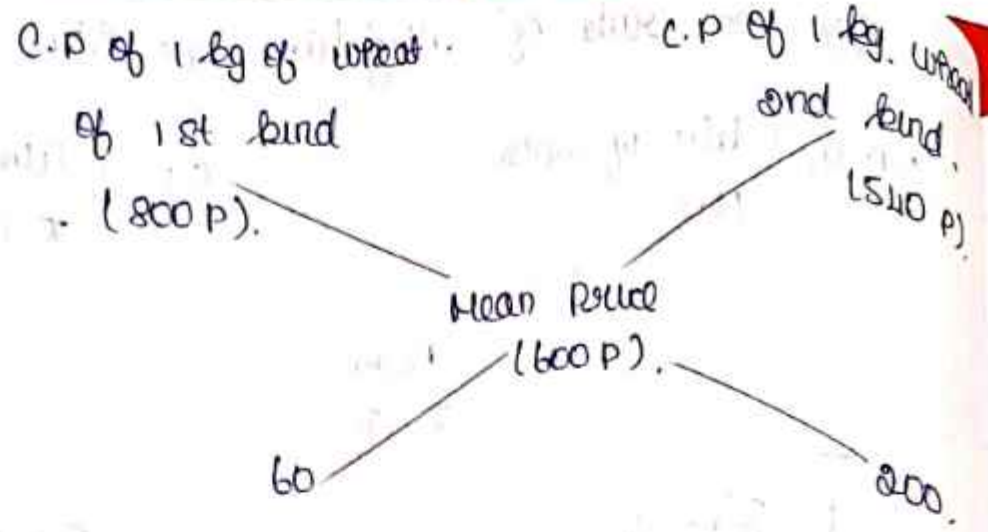
Gain = 20 %.

C.P of 1 kg mixture $= ₹\left(\frac{100}{120} \times 7.20\right)$

$= ₹. 6$

By the rule of allegation we have.

C.P of 1 kg of wheat of 1st kind C.P of



Wheat of 1st kind : wheat of 2nd kind

$= 60 : 200 \Rightarrow 3 : 10$

Let x kg of wheat 1st kind of mixed with 36 kg of wheat of 2nd kind

Then. $3 : 10 = x : 36$.

$10x = 3 \times 36$.

$x = 10.8$ kg

5) The milk and water in two vessels A & B are in the ratio of 4 : 3 and 2 : 3 respectively. In what ratio the liquid in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water?

Soln:

Let the CP of milk be ₹. 1 per litre

milk in 1 litre mixture of A $= \frac{4}{7}$ litre

milk in 1 litre mixture of B $= \frac{2}{5}$ litre

mixture in 1 litre mixture of C = $\frac{1}{2}$ litre

C.P of 1 litre mixture A = ₹. $\frac{11}{7}$.

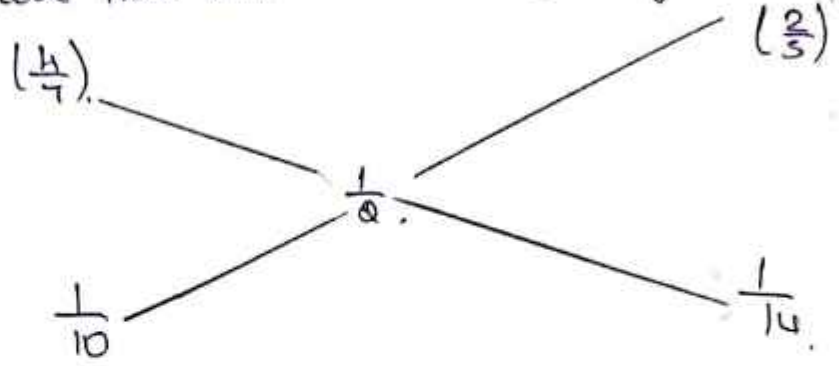
C.P of 1 litre mixture in B = ₹. $\frac{2}{5}$

Mean price = ₹. $\frac{1}{2}$.

By the rule of allgation we have

C.P of 1 litre mix in A

C.P of 1 litre mix in B



Required ratio.

$= \frac{1}{10} : \frac{1}{14}$

$= 7 : 5$